B.Sc./5th Sem (H)/MATH/22(CBCS)

2022

5th Semester Examination

MATHEMATICS (Honours)

Paper : C 11-T

[Partial Differential Equations and Applications]

[CBCS]

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer any ***ten*** questions : 2×10=20

   (a) What is ballistics? Write different types of ballistics.

   (b) Define quasi-linear and semi-linear partial differential equation.

   (c) Find the general solution of second order PDE
   $$4\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}=0.$$

   (d) What is the nature of the second order PDE
   $$\frac{\partial^2 z}{\partial y^2}-y\frac{\partial^2 z}{\partial x^2}+x^3 z=0?$$

P.T.O.

(e) Let $a, b \in \mathbb{R}$ be such that $a^2+b^2 \neq 0$. Then prove that the Cauchy problem $a\frac{\partial z}{\partial x}+b\frac{\partial z}{\partial y}=1$, $x, y \in \mathbb{R}$ with $z(x, y)=x$ on $ax+by=1$ has a unique solution.

(f) Find the characteristic curve of PDE : $2y\frac{\partial z}{\partial x}+\left(2x+y^2\right)\frac{\partial z}{\partial y}=0$ which is passing through the point (0, 0).

(g) Find the equations of the characteristic curves of the PDE $\left(x^2+2y\right)\frac{\partial^2 z}{\partial x^2}+\left(y^3-y+x\right)\frac{\partial^2 z}{\partial y^2}$ $+x^2(y-1)\frac{\partial^2 z}{\partial x \partial y}+3\frac{\partial z}{\partial x}+z=0$ which are passing through the point $x = 1, y = 1$.

(h) Let $z(x, t)$ be the equation of $\frac{\partial^2 z}{\partial x^2}=\frac{\partial^2 z}{\partial t^2}$ with $z(x, 0)=\cos(5\pi x)$ and $\frac{\partial z}{\partial t}(x, 0)=0$. Then prove that $z(1, 1) = 1$.

(i) Show that the solution of the PDE : $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=0$ is of the form $f(y/x)$.

(j) Prove that the partial differential equation $x\frac{\partial^2 z}{\partial x^2}+y\frac{\partial^2 z}{\partial y^2}=0$ is elliptic type for $x<0, y>0$.

(k) Find the two families of surfaces that generate the characteristics of $(3y-2z)\frac{\partial z}{\partial x}+(z-3x)\frac{\partial z}{\partial x}$ $=2x-y$.

(l) Find the partial differential equation by eliminating arbitrary constants a and b from $z=(x+a)(y+b)$.

(m) Define apsidal angle and apsidal distance.

(n) Prove that a planet has only a radial acceleration towards the Sun.

(o) Prove that at an apse on a central orbit, the velocity is proportional to the reciprocal of the radius vector.

Group - B

2. Answer any ***four*** questions : 5×4=20

(a) A particle moves with a central acceleration $\mu \div (\text{distance})^2$; it is projected with velocity v at a distance $\mathbb{R}$. Show that its path is a rectangular hyperbola if the angle of projection is

$$\sin^{-1}\left[\mu \Big/ \left\{VR\left(V^2-\frac{2\mu}{R}\right)^{1/2}\right\}\right].$$

P.T.O.

(b) A spherical raindrop falls through a cloud while accumulating mass at a rate λr^2 where r is its radius and $\lambda > 0$. Find its velocity at time t if it starts from rest with radius a.

(c) Find the integral surface of the PDE, $x(z+2a)p + (xz+2yz+2ay)q = z(z+a)$.

(d) Using the method of separation of variables, solve :

$\frac{\partial z}{\partial x} = q\frac{\partial z}{\partial t} + z$ where $z(x, 0) = 6e^{-3x}$.

(e) Reduce the wave equation $\frac{\partial^2 z}{\partial t^2} = c^2\frac{\partial^2 z}{\partial x^2}$ to canonical form.

(f) Solve $z^2 = pqxy$ by Charpit's method.

Group - C

3. Answer any *two* questions : 10×2=20

(a) (i) If a point moves on a Curve with constant tangential acceleration such that the magnitudes of the tangential velocity and normal acceleration are in a constant ratio, find the (s, ψ) equation of the curve.

(ii) Solve $(D^3 - 3DD'^2 - 2D'^3)z = \cos(x+2y)$.

(b) (i) A particle is projected with velocity V from

the cusp of a smooth inverted cycloid down the arc, show that the time of reaching the vertex is $2\sqrt{\frac{a}{g}}\tan^{-1}\left[\sqrt{\frac{4ag}{V}}\right]$. 4+6

(ii) Using the method of separation of variables, solve the following wave equation described by

PDE : $\frac{\partial^2 z}{\partial t^2} = 4\frac{\partial^2 z}{\partial x^2}$

BCS : $z(0, t) = 0,\ z(s, t) = 0$

ICS : $z(x, 0) = 0,\ \left(\frac{\partial z}{\partial t}\right)_{t=0} = 5\sin \pi x$. 5+5

(c) (i) Solve the boundary value problem $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$, $u(x, 0) = 20$, $u(0, t) = 0$, $u(L, t) = 0$.

(ii) Find the integral surface of the linear PDE $2y(z-3)\frac{\partial z}{\partial x} + (2x-z)\frac{\partial z}{\partial y} = y(2x-3)$ which passes through the circle $x^2 + y^2 = 2x,\ z = 0$. 5+5

P.T.O.

(d) (i) Solve two dimensional Laplace's equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

with BCS $z(0, y) = 0,\ z(l, y) = 0$

and $z(x, y) \to 0$ as $y \to \infty$

$z(x, 0) = f(x)$

(ii) Let $u\,(x, y)$ be the solution of the Cauchy problem $\frac{\partial u}{\partial y} - x\frac{\partial u}{\partial x} + u = 1$, where $-\infty < x < \infty,\ y \geq 0$ and $u\,(x, 0) = \sin x$, then find $u(0, 1)$. 5+5