বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. Honours Examinations 2022

(Under CBCS Pattern)

Semester - II

Subject : MATHEMATICS

Paper : GE 2 - T

[ALGEBRA]

Full Marks : 60

Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

1. Answer any ***five*** questions : 2×5=10

(a) If *a, b, c* be positive real numbers, prove that $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b} > \frac{3}{2}$ unless $a=b=c$.

(b) Prove that for a complex number *z*, $|z| \ge \frac{1}{\sqrt{2}}\left[|\mathrm{Re}(z)|+|\mathrm{Im}\,g(z)|\right]$.

(c) Discuss the maximum number of complex root of $x^6-3x^2-2x-3=0$ using Descartes' rule of sign.

(d) Solve the equation $x^4+x^2-2x+6=0$ where it is given that $(1+i)$ is a root.

(e) Prove that the product of any *m* consecutive integers is divisible by *m*.

(f) A set contains only null vector. Is it independent? Exlpain.

(g) Let P be an orthogonal matrix with det $(P) = -1$. Prove that -1 is an eigen value of P.

(h) If $a \mid b$, with $gcd\ (a,b) = 1$, then show that $a \mid c$.

2. Answer any ***four*** questions : 5×4=20

(a) State and prove Cayley-Hamilton theorem.

(b) Find the basis and the dimension of the subspace W of R^3 where $W = \{(x, y, z) \in R^3 : x + y + z = 0\}$.

(c) Show that the intersection of two subspaces of a vector space V over a field F is a subspace of V. Is the union of two subspaces a subspace of V ? If not, discuss the condition.

(d) Determine the conditions for which the system : $x + y + z = b;\ 2x + y + 3z = b + 1$; $5x + 2y + az = b^2$ admits of (i) unique solution, (ii) no solution and (iii) many solutions.

(e) A linear mapping $T : R^3 \to R^3$ is defined by

$T(x_1, x_2, x_3) = (2x_1 + x_2 - x_3,\ x_2 + 4x_3,\ x_1 + x_2 + 3x_3), (x_1, x_2, x_3) \in R^3$, Find the matrix relative to the order basis $\{(1,0,0),(0,1,0),(0,0,1)\}$ of R^3.

(f) Show that the eigen values of a real symmetric matrix are all real.

3. Answer any ***three*** questions : 10×3=30

(a) (i) If n be a positive integer, prove that

$$\frac{1}{2\sqrt{n+1}} < \frac{1.3.5..........(2n-1)}{2.4.6........2n} < \frac{1}{\sqrt{2n+1}}$$ 5

(ii) Prove that the minimum value of $x^2+y^2+z^2$ is $\left(\frac{c}{7}\right)^2$ where x, y, z are positive real numbers subject to the condition $2x+3y+6z=c$, c being a constant. Find the values of x, y, z for which the minimum value is attained. 5

(b) (i) If $\cos\alpha+\cos\beta+\cos\gamma=0$ and $\sin\alpha+\sin\beta+\sin\gamma=0$ then prove that $\cos 3\alpha+\cos 3\beta+\cos 3\gamma=3\cos(\alpha+\beta+\gamma)$. 3

(ii) Show that the roots of $(1+z)^{2n}+(1-z)^{2n}$ are the values of $\pm i\tan\frac{(2r-1)\pi}{4n}, r=1,2,\ldots\ldots,n$. 5

(iii) Find the value of $\sqrt[n]{i}+\sqrt[n]{-i}$. 2

(c) (i) Find the integers u and v such that $63u+55v=1$. 2

(ii) Prove that for all integers $n>2$, n^3-1 is composite. 2

(iii) Establish that the difference of two consecutive cubes is never divisible by 2. 2

(iv) If $f: A\to B$ and $g: B\to C$ are both injective mappings then show that the composite mapping $g\circ f: A\to C$ is injective. 2

(v) If $gcd\,(a, b) = 1$, then $gcd\,(a^2, b^2) = 1$. 2

(d) (i) Solve the equation by Cardan's method : $x^3+9x^2+15x-25=0$. 6

(ii) The roots of the equation $x^3+px^2+qx+r=0\,(r\neq 0)$ are α,β,γ. Find the equation whose roots are $\alpha\beta-\gamma^2,\ \beta\gamma-\alpha^2,\ \gamma\alpha-\beta^2$. 4

(e) (i) Consider a set $\boldsymbol{Z}$ in which the relation ρ is defined by $a\rho b$ iff $3a+4b$ is divisible by 7. Examine whether ρ is an equivalence relation. 5

(ii) Let S be a real skew symmetric matrix of order n, then prove that $(I_n+S)^{-1}(I_n-S)$ is orthogonal. 5